PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If L is a finite extension of k and if k is a finite extension of F, then prove that L is a finite extension of F.

17. If $p(x)$ is a polynomial in $F(x)$ of degree $n \geq 1$ and is irreducible over F, then prove that there is an extension E of F, such that $[E:F]=n$, in which $p(x)$ has a root.

18. If K be a normal extension of F and if H be a subgroup of $G(K,F)$; and if $K_H = \{x \in k / \sigma(x) = x$ for all $\sigma \in H\}$ be the fixed field of H. Then prove that

    (a) $[K:K_H]=O(H)$

    (b) $H=G(K,K_H)$

19. Prove the a finite division ring is necessarily a commutative field.

20. If D be a division ring algebraic over F, the field of real numbers. Then prove that D is isomorphic to one off; the field of real numbers, the field of complex numbers, or the division ring of real quaternions.

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NOVEMBER/DECEMBER 2024

GMA21/DMA21 — ALGEBRA - II

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define algebraic over F.
2. Define algebraic number.
3. Define splitting field over F.
4. Define derivative of $f(x)$.
5. Define fixed field of G.
6. Define Galois Group of $f(x)$.
7. Define Solvable Group.
8. Define Euler Criterion.
9. Define algebraic over a field F for a division algebra D.
10. Define norm of x.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

Or

(b) If $m>0$ and n are integers, prove that $e^{m/n}$ is transcendental.

12. (a) If $f(x)\in F[x]$, then prove that there is a finite extension E of F in which $f(x)$ has a root.

Or

(b) If $f(x)\in F[x]$ is irreducible, then prove the following :

(i) If the characteristic of F is 0, has multiple roots.

(ii) If the characteristic of F is $p\neq 0$, $f(x)$ has a multiple roots only if it is of the form $f(x)=g(x^p)$.

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13. (a) Prove that $G(K,F)$ is a subgroup of the group of all automorphisms of K.

Or

(b) Prove that the elementary symmetric functions in $x_1,...,x_n$ are indeed symmetric functions in $x_1,...x_n$.

14. (a) If $G=S_n$ where $n\geq 5$; then prove that $G^{(k)}$ for $K=1,2,...$ contains every Three-cycles of S_n.

Or

(b) Prove that for every prime number p and every positive integer m there exists a field having p^m elements.

15. (a) If C be the filed of complex numbers and suppose that the division ring D is algebraic over C. Then prove that $D=C$.

Or

(b) For all $x, y\in Q$, prove that $N(xy)=N(x)N(y)$.